Flavor and Spin Contents of the Nucleon in the Quark Model with Chiral Symmetry

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A simple calculation in the framework of the chiral quark theory of Manohar and Georgi yields results that can account for many of the "failures" of the naive quark model: significant strange quark content in the nucleon as indicated by the value of $\sigma_{\pi N}$, the $\overline{u}$-$\overline{d}$ a symmetry in the nucleon as measured by the deviation from the Gottfried sum rule and by the Drell-Yan process, as well as the various quark contributions to the nucleon spin as measured by deep inelastic polarized lepton-nucleon scattering.

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One of the outstanding problems in nonperturbative QCD is to understand, at a more fundamental level, the successes and failures of the nonrelativistic quark model. An important step in that direction was taken several years ago by Manohar and Georgi [1] when they presented their chiral quark model with an effective Lagrangian for quarks, gluons, and Goldstone bosons in the region between the chiral symmetry breaking and the confinement scales. They demonstrated how the successes of the simple quark model could be understood in this framework, which naturally suggests a chiral symmetry breaking scale $\Lambda_{\chi \mathrm{SB}} \simeq 1$ GeV, significantly higher than the QCD confinement scale. And it also allows for the possibility for a much reduced (effective) strong coupling α_S, thus the result of hadrons as weakly bound states of constituent quarks. A meaningful calculation of the magnetic moments of the baryons (with great success) can also be carried out.

In this Letter we shall show that this framework, with a simple extension, can also remedy many of the "experimental failures" of the simple quark model. It has been known for a long time that the measured value of the pion nucleon sigma term $\sigma_{\pi N}$ indicates a significant strange quark presence in the nucleon [2]. This failure of the simple quark model is further highlighted when the deep inelastic polarized muon-proton scattering measurements made by the European Muon Collaboration (EMC) [3] indicated a significant contribution to the proton spin by the strange quarks in the sea [4]. Then along came the results from the New Muon Collaboration (NMC) [5] showing that the Gottfried sum rule [6] is not satisfied experimentally, indicating a nucleon sea that is quite asymmetric with respect to its $\overline{u}$ and $\overline{d}$ quark contents. This basic piece of physics is now confirmed by a dedicated Drell-Yan experiment colliding protons on proton and on neutron targets [7].

An important feature of the chiral quark model is that the internal gluon effects can be small compared to the internal Goldstone bosons and quarks. As we shall demonstrate, this picture can account for, in terms of two parameters, the broad pattern of the observed flavor and spin contents of the nucleon. In the chiral quark model, the dominant process is the fluctuation of quark into quark plus a Nambu-Goldstone boson [1,8–10]. This basic interaction causes a modification of the spin content because a quark can change its helicity by emitting a spin zero meson, Fig. 1(a). It causes a modification of the flavor content because the quark sea brought on by Goldstone boson fluctuations is, unlike that due to gluon emissions, flavor dependent, Fig. 1(b). Instead of using the parton evolution equation of the chiral quark theory, as has been carried out by Eichten, Hinchliffe, and Quigg [10], we will illustrate the basic physical mechanism with a schematic calculation, as was also considered by these authors.

In the absence of interactions, the proton is made up of two u quarks and one d quark. We now calculate the proton's flavor content after any one of these quarks emits

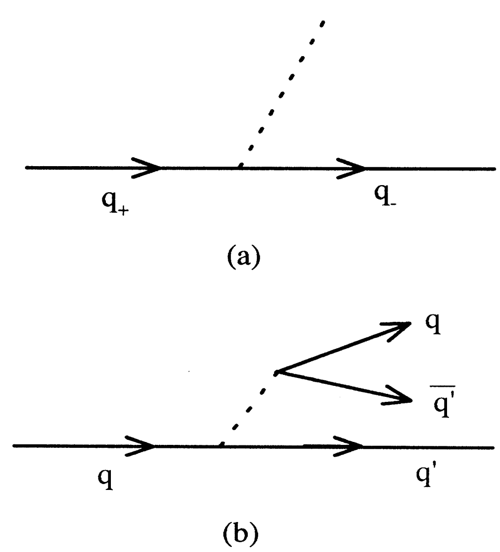

FIG. 1. Spin and flavor corrections due to Goldstone boson fluctuations. (a) A spin zero meson couples to quarks of opposite helicites. (b) Production of a $(q'\overline{q'})$ pair via Goldstone emission.

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a quark-antiquark pair via (virtual) Goldstone bosons, which have interaction vertices $\mathcal{L}_I = g_8 \overline{q}\tilde{\phi}q$,

$$\tilde{\phi} = \sum_{i=1}^{8} \lambda_i \phi_i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix},$$

$q = (u, d, s)$, and the λ's are the Gell-Mann matrices. We suppressed all the space-time structure and have only displayed the flavor content of the coupling. The probability amplitudes of meson emission from a u quark to various meson-quark states are

$$\Psi(u \longrightarrow \pi^+ d) = \sqrt{2}\,\Psi(u \longrightarrow \pi^0 u) = g_8, \text{ etc.} \tag{1}$$

From this, we deduce that $1 - \frac{8}{3}a$ is the probability of no meson emission with $a \propto |g_8|^2$ being the probability of emitting a π^+, etc. By substituting the quark content of the mesons into the above equation and squaring the amplitude, one obtains the proton's flavor composition (after one interaction)

$$(1 - \tfrac{8}{3}a)(2\hat{u} + \hat{d}) + 2|\Psi(u)|^2 + |\Psi(d)|^2,$$

with

$$\begin{aligned} |\Psi(u)|^2 &= \tfrac{2}{9}[14\hat{u} + 2\hat{\overline{u}} + 5(\hat{d} + \hat{\overline{d}} + \hat{s} + \hat{\overline{s}})]a, \\ |\Psi(d)|^2 &= \tfrac{2}{9}[14\hat{d} + 2\hat{\overline{d}} + 5(\hat{u} + \hat{\overline{u}} + \hat{s} + \hat{\overline{s}})]a, \end{aligned} \tag{2}$$

where the q quark number density is given by the coefficient in front of the symbol $\hat{q}$. In this picture the strange quarks in the sea are brought about the fluctuation of the valence u and d quarks into the s-quark-containing mesons K's and η. An asymmetry develops between $\overline{u}$ and $\overline{d}$ distributions because there is an initial u-d asymmetry in the valence quarks and the u quark cannot fluctuate into π^- (hence a final state containing $\overline{u}$) while the d quark cannot fluctuate into π^+, etc.

Since the $1/N_c$ expansion, N_c being the number of colors, is thought to be a useful approximation scheme for QCD, it will be worthwhile to see what it will teach us here. The leading contribution comes from the planar diagrams. At this order, there are *nine* Goldstone bosons, including the unmixed diagonal channels $\overline{u}u$, $\overline{d}d$, and $\overline{s}s$, all with the same couplings. If we express this in the language of SU(3), besides the octet there is also the singlet, with the singlet Yukawa coupling being equal to the octet coupling $g_0 = g_8$. When this ninth singlet η' meson (here it is not the physical η', but the singlet meson in the planar approximation) is included in the computation, one finds a surprising result of *a flavor-independent sea* [10]. Namely, the original $\overline{u}$-$\overline{d}$ asymmetry due to the asymmetric $\pi^{\pm}$ fluctuations is canceled by the corresponding asymmetry due to the coherent neutral meson emissions. As a result, there is now an equal number of $u\overline{u}$ and $d\overline{d}$ as well as $s\overline{s}$ pairs.

Mathematically, this flavor independence comes about as follows. Equating the coupling constants $g_8 = g_0$ in the extended vertex

$$\mathcal{L}_I = g_8 \sum_{i=1}^{8} \overline{q}\lambda_i\phi_i q + \sqrt{\frac{2}{3}}\, g_0 \overline{q}\eta' q \tag{3}$$

and squaring the amplitude, one obtains the probability distribution of

$$\sum_{i=1}^{8} (\overline{q}\lambda_i q)(\overline{q}\lambda_i q) + \frac{2}{3}(\overline{q}q)(\overline{q}q),$$

which has the index structure as

$$\sum_{i=1}^{8} (\lambda_i)_{ab}(\lambda_i)_{cd} + \frac{2}{3}\delta_{ab}\delta_{cd} = 2\delta_{ad}\delta_{bc}.$$

The right-hand side, deduced from a well-known identity of the Gell-Mann matrices [11], clearly shows the flavor independent nature of the result.

While the mathematics of this flavor asymmetry cancellation is fairly straightforward, the physics is more intriguing. It shows that the deviation from an SU(3) symmetric sea should, for the most part, spring from the subleading contributions in the $1/N_c$ expansion. In the more descriptive "topological expansion" language used, for example, by Veneziano [12], this corresponds to the nonplanar contributions, which are known to be important for an adequate description of the nonperturbative QCD. In fact, the axial anomaly vanishes at the planar-diagram level. The resolution of the η' mass problem depends on the nonplanar physics. However, with the admission of the nonplanar contributions, the equality between the octet and singlet couplings is broken, $g_0/g_8 \equiv \zeta \neq 1$. We can now repeat the above calculation and express our result for flavor densities in terms of the two parameters—this coupling ratio ζ and a, the probability of π^+ emission $u = 2 + \overline{u}$, $d = 1 + \overline{d}$, and $s = \overline{s}$, with the antiquark contents being

$$\overline{u} = \tfrac{a}{3}(\zeta^2 + 2\zeta + 6), \quad \overline{d} = \tfrac{a}{3}(\zeta^2 + 8), \tag{4}$$

$$\overline{s} = \tfrac{a}{3}(\zeta^2 - 2\zeta + 10). \tag{5}$$

Let us review our phenomenological knowledge of the flavor content of the nucleon: The deviation from the Gottfried sum rule for deep inelastic lepton-nucleon scattering is interpreted as showing an asymmetry between the $\overline{u}$ and $\overline{d}$ quarks in the nucleon sea,

$$\left[\int_0^1 dx\, \frac{F_2^p(x) - F_2^n(x)}{x} - \frac{1}{3}\right] = \frac{2}{3}(\overline{u} - \overline{d}). \tag{6}$$

The NMC measurements did indeed show the Gottfried integral being significantly different from one-third,

$$I_G = 0.235 \pm 0.026. \tag{7}$$

Here we have quoted the new NMC result published this year [5].

This $\overline{u}$-$\overline{d}$ asymmetry has now been confirmed by the NA51 Collaboration [7] at CERN in a Drell-Yan experiment of scattering protons on proton and on deuteron targets [13]. The measured ratio of muon pair production cross sections σ_{pp} and σ_{pn} can be expressed as the antiquark content ratio

$$\overline{u}/\overline{d} = 0.51 \pm 0.04(\text{stat}) \pm 0.05(\text{syst}). \tag{8}$$

This quantity is of particular interest in our model calculation, since it depends only on one parameter

$$\overline{u}/\overline{d} = \frac{\zeta^2 + 2\zeta + 6}{\zeta^2 + 8}. \tag{9}$$

Interestingly, with only the octet Goldstone contribution (thus $\zeta = 0$) this ratio is fixed to be 0.75, comparing rather poorly with the experimental result in Eq. (8). With the inclusion of the singlet contribution ($\zeta \neq 0$), this expression still implies a lower bound for the $\overline{u}/\overline{d}$ ratio of $\frac{1}{2}$ at $\zeta = -2$, and an upper bound of $\frac{5}{3}$ at $\zeta = 4$. More relevantly, the experimental value in Eq. (8) implies that the coupling ratio ζ must be *negative*: $-4.3 \le \zeta \le -0.7$. Given the crudity of our calculation and the sensitivity of the quadratic relations, we shall merely illustrate our model calculation results in the subsequent discussion with the following simple parameter choice:

$$a = 0.1 \quad \text{and} \quad \zeta = -1.2, \tag{10}$$

which corresponds to $\overline{u}/\overline{d} = 0.53$, and to fixing a in Eq. (4) so that Eq. (6) will yield the central experimental value of the Gottfried sum

$$I_G = \tfrac{1}{3} + \tfrac{2}{3}(\overline{u} - \overline{d}) = \tfrac{1}{3} + \tfrac{4}{9}(\zeta - 1)a = 0.235.$$

The fractions of quark flavors $f_a \equiv (q_a + \overline{q}_a)/\sum(q + \overline{q})$ in the proton can also be calculated from Eq. (4) with the parameters of Eq. (10),

$$f_u \simeq 0.48, \quad f_d \simeq 0.33, \quad f_s \simeq 0.19. \tag{11}$$

Observationally [14], the strange quark fraction f_s can be deduced from the phenomenological value [15] of the πN sigma term

$$\sigma_{\pi N} = \hat{m}\langle N|\overline{u}u + \overline{d}d|N\rangle = 45 \pm 10 \text{ MeV}, \tag{12}$$

where $\hat{m} \equiv \frac{1}{2}(m_u + m_d)$, and the SU(3) symmetry relation

$$\begin{aligned} M_8 &\equiv \tfrac{1}{3}(\hat{m} - m_s)\langle N|\overline{u}u + \overline{d}d - 2\overline{s}s|N\rangle \\ &= M_\Lambda - M_\Xi \simeq -200 \text{ MeV}. \end{aligned} \tag{13}$$

Using the current algebra result of $m_s/\hat{m} = 25$, we obtain the ratio

$$1 = 2y \equiv \frac{\langle \overline{u}u + \overline{d}d - 2\overline{s}s\rangle}{\langle \overline{u}u + \overline{d}d\rangle} = \frac{3\hat{m}M_8}{(\hat{m} - m_s)\sigma_{\pi N}} \simeq \frac{5}{9}.$$

Keeping in mind that the scalar operator $\overline{q}q$ measures the sum of the quark and antiquark numbers (as opposed to $q^\dagger q$, which measures the difference), we find

$$(f_s)_{\sigma_{\pi N}} = \frac{y}{1 + y} \simeq 0.18. \tag{14}$$

We now turn to the nucleon spin content. In the no interaction limit, the spin-up proton (p_+) state $|p_+\rangle = \frac{1}{\sqrt{6}}2|u_+u_+d_-\rangle - |u_+u_-d_+\rangle - |u_-u_+d_+\rangle$ implies that the probabilities of finding u_+ (u quark in the spin-up state), u_-, d_+, and d_- in p_+ are $\frac{5}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, and $\frac{2}{3}$, respectively. These values will be altered by the same meson emission processes of the chiral quark model, as discussed above in connection with proton's flavor content. The total probability of Goldstone emission being $\frac{a}{3}(\zeta^2 + 8)$, the contributions of the various spin states, after one interaction, can then be read off from

$$\begin{aligned} &[1 - \tfrac{a}{3}(\zeta^2 + 8)](\tfrac{5}{3}\widehat{u_+} + \tfrac{1}{3}\widehat{u_-} + \tfrac{1}{3}\widehat{d_+} + \tfrac{2}{3}\widehat{d_-}) \\ &\quad + \tfrac{5}{3}|\Psi(u_+)|^2 + \tfrac{1}{3}|\Psi(u_-)|^2 + \tfrac{1}{3}|\Psi(d_+)|^2 + \tfrac{2}{3}|\Psi(d_-)|^2, \end{aligned}$$

where amplitudes $\Psi(q_\pm)$ can be calculated in a similar manner as that done for Eq. (2). The quark contributions to the proton spin $\Delta q = q_+ - q_- + \overline{q}_+ - \overline{q}_-$ are

$$\Delta u = \tfrac{4}{3} - \tfrac{1}{9}(8\zeta^2 + 37)a, \tag{15}$$

$$\Delta d = -\tfrac{1}{3} + \tfrac{2}{9}(\zeta^2 - 1)a, \tag{16}$$

$$\Delta s = -a, \tag{17}$$

which for the parameters of Eq. (10) yields the values

$$\Delta u = 0.79, \quad \Delta d = -0.32, \quad \Delta s = -0.10. \tag{18}$$

We note parenthetically that our calculation can be looked up as an explicit realization of the renormalization effect due to Goldstone boson loops in the chiral quark model as discussed, for example, in [9].

Since the 1988 announcement by EMC of their proton spin content result, there have been significant new experimental developments in the measurement of the spin-dependent structure functions of the neutron, as well as that of the proton [16]. In the meantime, further higher order perturbative QCD calculations have been carried out for such spin-dependent structure function sum rules [17]. Taking into account these higher order results [hence the perturbative QCD predicted Q^2 dependence through $\alpha_s(Q^2)$], Ellis and Karliner have recently shown [18] that all the diverse experimental measurements are consistent with each other, and that the fundamental Bjorken sum rule is verified to about 12%. When the final result, after using a flavor SU(3) symmetry relation [similar to that of Eq. (13)], is expressed in terms of the individual quark contributions to the proton spin

$$\Delta u_{\text{exp}} = 0.83, \quad \Delta d_{\text{exp}} = -0.42, \quad \Delta s_{\text{exp}} = -0.10, \tag{19}$$

we have a comparison with our result in Eq. (18). One notes that the total quark spin contribution actually does not vanish, $\Delta\sum \equiv \Delta u + \Delta d + \Delta s \simeq 0.31$, which is to be compared to our model calculation result of 0.37 in Eq. (18).

We should also note that ours is an SU(3) *symmetric* computation. The basic feature that the strange quark is heavier than the up and down quarks have not been taken into account. The meson emission corrections for each component of an SU(3) multiplet must, in such a calculation, be the same. Consequently, the naive quark model ratio of $\Delta_3/\Delta_8 = \frac{5}{3}$ is unchanged (where

$\Delta_3 = \Delta u - \Delta d$ and $\Delta_8 = \Delta u + \Delta d - 2\Delta s$). It is about 25% lower than the phenomenological value [19] of $(\Delta_3/\Delta_8)_{\rm exp} \simeq 2.1$. And, the similarly defined flavor-fraction ratio of $f_3/f_8 = \frac{1}{3}$ is greater than the experimental value of $(f_3/f_8)_{\rm exp} \simeq 0.23$. Inclusion of the SU(3) breakings will necessarily increase the number of parameters and complicate the model calculations. We postpone such an endeavor to a later stage, and choose to present our principal results without having them masked by this complication.

Overall we find it rather encouraging that this simple calculation in a theoretically well-motivated framework can account for many of the puzzling features discovered in recent years of the spin and flavor contents of the nucleon. To us what is significant is this broad pattern of agreement in one unified calculation. Our effort overlaps with many of the previous works [20], where these effects have been discussed as unrelated phenomena. What we wish to emphasize in this work is that the nonrelativistic quark model, when supplemented with the Goldstone structure, does yield an adequate approximation of the observed low energy physics. A key ingredient in this implementation is the inclusion of the ninth Goldstone boson with a differently renormalized coupling $g_0 \simeq 1.2g_8$. Presumably the success with such an inclusion shows that, above the confinement scale, the nonplanar mechanism which endows the η' with a mass can still be treated as a perturbation as suggested by the $1/N_c$ expansion of QCD.

With this first encouraging result, it might be worthwhile to embark on a more elaborate field theoretical calculation. This will involve more mass, cutoff parameters, and further phenomenological inputs, but it will also allow a more detailed comparison of the Q^2 and x dependences of the structure functions between the chiral quark theory expectations and the experimental measurements.

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[1] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979), Sec. 6; H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, Menlo Park, CA, 1984), Sec. 6.4 and 6.5.

[2] T.P. Cheng, Phys. Rev. D **13**, 2161 (1976); T.P. Cheng and R.F. Dashen, Phys. Rev. Lett. **26**, 594 (1971).

[3] European Muon Collaboration, J. Ashman *et al.,* Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1990).

[4] J. Ellis and R.L. Jaffe, Phys. Rev. D **9**, 1444 (1974).

[5] New Muon Collaboration, P. Amaudruz *et al.,* Phys. Rev. Lett. **66**, 2712 (1991); M. Arneodo *et al.,* Phys. Rev. D **50**, R1 (1994).

[6] K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).

[7] NA51 Collaboration, A. Baldit *et al.,* Phys. Lett. B **332**, 244 (1994).

[8] S.J. Brodsky, J. Ellis, and M. Karliner, Phys. Lett. B **206**, 309 (1988); H. Fritzsch, *ibid.* **229**, 122 (1989); **256**, 75 (1991).

[9] D.B. Kaplan and A. Manohar, Nucl. Phys. **B310**, 527 (1988).

[10] E.J. Eichten, I. Hinchliffe, and C. Quigg, Phys. Rev. D **45**, 2269 (1992); see also J.D. Bjorken, Report No. SLAC-PUB-5608, 1991 (unpublished).

[11] See, for example, T.P. Cheng and Ling-Fong Li, *Gauge Theory of Elementary Particle Physics* (Clarendon, Oxford, 1984), p. 110.

[12] G. Veneziano, Nucl. Phys. **B159**, 213 (1979); **B117**, 519 (1977).

[13] S.D. Ellis and W.J. Stirling, Phys. Lett. B **256**, 258 (1993).

[14] We are not able to compare the strange quark content as measured in neutrino charm production experiments because that quantitatively involves the first moment of the strange quark distribution $x[s(x) + \bar{s}(x)]$ while our schematic calculation only yields results *averaged* (unweighted) over all momentum fractions x.

[15] J. Gasser, H. Leutwyler, and M.E. Sainio, Phys. Lett. B **253**, 252 (1991).

[16] Spin Muon Collaboration, B. Adeva *et al.,* Phys. Lett. B **302**, 533 (1993); D. Adams *et al.,* Phys. Lett. B **329**, 399 (1994); E142 Collaboration, P.L. Anthony *et al.,* Phys. Rev. Lett. **71**, 959 (1993); E143 Collaboration, R. Arnold *et al.,* in the Proceedings of the St. Petersburg (Florida) Conference, June, 1994 (unpublished).

[17] J. Kodaira *et al.,* Phys. Rev. D **20**, 627 (1979); S.A. Larin, F.V. Tkachev, and J.A.M. Vermaseren, Phys. Rev. Lett. **66**, 862 (1991); A.L. Kataev and V. Starshenko, CERN Reports No. Th.7208-94 and No. hep-ph/9403383.

[18] J. Ellis and M. Karliner, Phys. Lett. B **341**, 397 (1995); see also F. Close and R.G. Roberts, Phys. Lett. B **316**, 165 (1993).

[19] $\Delta_8 = 0.601 \pm 0.038$ from S.Y. Hsueh *et al.* [Phys. Rev. D **38**, 2056 (1988)]; $\Delta_3 = g_A = 1.2573 \pm 0.0028$.

[20] For the relevant prior investigations, besides those already cited, see, e.g., T. Hatsuda and T. Kunihiro [Phys. Rep. (to be published)]; for $\sigma_{\pi N}$ and the strange quark content, see J.F. Donoghue and C.R. Nappi [Phys. Lett. **168B**, 105 (1986)]; for the proton spin contents, see, e.g., H. Dreiner, J. Ellis, and R. Flores [Phys. Lett. B **221**, 169 (1989)]; S. Forte, Nucl. Phys. **B331**, 1 (1990); for the Gottfried sum rule, see, e.g., E.M. Henley and G.A. Miller [Phys. Lett. B **251**, 453 (1990)]; S. Forte, Phys. Rev. D **47**, 1842 (1993); R. Ball and S. Forte, Nucl. Phys. **B425**, 516 (1994).