Nonconservation of Separate μ- and e-Lepton Numbers in Gauge Theories with $V+A$ Currents

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In gauge theories with $V+A$ currents and new leptons, effects that do not separately conserve the μ- and e-lepton numbers are dramatically larger than those predicted in the pure $V-A$ theories. The branching ratios for $\mu\to e\gamma$ and $K_L\to e\mu$ are found to be comparable to their present experimental limits.

The occurrence of $\mu\to e\gamma$, $\mu\to ee\bar{e}$, $K_L\to e\bar{\mu}$, or $K\to\pi e\bar{\mu}$ decays would signify that the muon and electron lepton numbers are not separately conserved. We report here the result of our investigation in gauge theories of such rare weak processes,[1] as well as the related phenomenon of neutrino oscillations $\nu_e\leftrightarrow\nu_\mu$.

In gauge theories for which the mass eigenstates are generally not eigenstates with respect to weak interactions, we would not expect separate conservations of μ- and e-lepton numbers. They may nevertheless hold if the theory has some extra degeneracy. Consider the standard $SU(2)\otimes U(1)$ gauge theory with only $V-A$ couplings

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L. \tag{1}$$

If neutrino masses are degenerate (in particular, zero), the "mixing" between neutrinos,

$$\nu_e=\nu_1(\theta)\equiv(\cos\theta)\nu_1+(\sin\theta)\nu_2,$$
$$\nu_\mu=\nu_2(\theta)\equiv-(\sin\theta)\nu_1+(\cos\theta)\nu_2,$$

would be meaningless, and we still have separately conserved μ- and e-lepton numbers. However, in this pure $V-A$ theory, even if neutrinos have different masses, all nonconservation effects will still be extremely suppressed, being always proportional to the difference between squares of the neutrino masses $\Delta m_\nu^2=m_{\nu_1}^2-m_{\nu_2}^2$. In other words, on a typical mass scale neutrinos are, for all practical purposes, massless—hence degenerate.

Recently many authors have suggested that new leptons, and new quarks beyond charm, exist and that some of their weak couplings are $V+A$ in nature. The observed $V-A$ character of the weak interactions is supposed to break down when new degrees of freedom are excited at high energies. However, rare decays such as $\mu\to e\gamma$ and $K_L\to e\mu$, being higher-order weak processes, can also probe the structure of weak interactions on an energy scale characterized by the W-boson mass M_W. Thus the new features of weak-interaction theories will also manifest themselves in these rare "low-energy" processes.

To be specific, consider the following simple theory[2]:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}N_e\\ e\end{pmatrix}_R \begin{pmatrix}N_\mu\\ \mu\end{pmatrix}_R\cdots \tag{2}$$

with $\nu_e=\nu_1(\theta)$, $\nu_\mu=\nu_2(\theta)$, $N_e=N_1(\varphi)$, and $N_\mu=N_2(\varphi)$. $N_{1,2}$ are heavy-lepton mass eigenstates and should have masses in the GeV range. Now separate μ- and e-lepton-number nonconservation amplitudes will be proportional to $\Delta m_N^2=m_{N_1}^2-m_{N_2}^2$, which is probably on the order of 1 GeV^2. Thus the theory in (2) would predict decay rates for $\mu\to e\gamma$ and $K_L\to e\mu$ much larger than those by the $V-A$ theory of (1), by a factor[3] of $(\Delta m_N^2/\Delta m_\nu^2)^2\gtrsim10^{33}$.

$\mu\to e\gamma$ decay.—The dominant amplitude corresponds to the two $(V+A)(V+A)$ graphs $N=N_1$ and N_2 in Fig. 1. To leading order in $\Delta m_N^2/M_W^2$, we

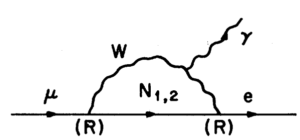

FIG. 1. One-loop diagram for the $\mu\to e\gamma$ decay meditated by neutral leptons. R stands for the $V+A$ coupling. W's are the gauge bosons, with unitary gauge propagators. We do not explicitly display diagrams with photon emission from external charged lines. They contribute only to the $\bar{e}\gamma_\lambda\mu\epsilon^\lambda$ amplitude which vanishes because of current conservation.

obtain

$$T_{\mu\to e\gamma} = e\left(\frac{g^2}{8M_W^2}\right)\frac{m_\mu}{32\pi^2}\cos\varphi\sin\varphi \times\left[\frac{m_{N_1}^2 - m_{N_2}^2}{M_W^2}\right]\bar{e}(1-\gamma_5)\sigma_{\lambda\nu}q^\nu\mu\epsilon^\lambda. \quad (3)$$

The $(V-A)(V-A)$ diagrams with N replaced by neutrinos, being proportional to $\Delta m_\nu^2/M_W^2$, are totally negligible. Equation (3) yields a decay rate of

$$\Gamma_{\mu\to e\gamma} = (3\alpha/32\pi)(\cos\varphi\sin\varphi\,\Delta m_N^2 M_W^{-2})^2\Gamma_\mu, \quad (4)$$

where $\Gamma_\mu \simeq G_F^2 m_\mu^5/192\pi^3$ is the $\mu\to e\bar{\nu}_e\nu_\mu$ decay width. For $(\cos\varphi\sin\varphi\,\Delta m_N^2)^2 \simeq 1\ \mathrm{GeV}^4$ and $M_W \simeq 50$ GeV we get $\Gamma_{\mu\to e\gamma}/\Gamma_\mu \simeq 10^{-10}$. This is to be compared with the present experimental limit of 2.2×10^{-8} set well over ten years ago.[4]

The amplitude in Eq. (3) is $O(eG_F M_W^{-2})$. This result may be understood with the following heuristic argument. The leading terms in the $(V+A)(V+A)$ amplitudes for each of the diagrams $(N=N_1, N_2,\dots)$ are independent of heavy-lepton masses, and they cancel.[5] For the next leading terms, which are proportional to $m_{N_i}^2$, we can legitimately approximate the W-boson propagators by $(q^2-M_W^2)^{-2}\sim M_W^{-4}$, since the remaining loop integral will still be convergent.

$K_L\to e\bar{\mu}$ decay.—The free-quark graphs are represented by Fig. 2. There is a four-way cancellation among graphs with uN_1, cN_1, uN_2, and cN_2 fermion exchanges: the lepton N_i cancellation (same as the one discussed in $\mu\to e\gamma$ decay) plus the Glashow-Iliopoulos-Maiani (GIM) cancellation.[6,7] The resultant amplitude is

$$T_{\bar{d}s\underset{2W}{\to}\bar{\mu}e} \simeq \frac{g^4}{16\pi^2}\frac{\epsilon}{M_W^2} \times\{\bar{d}\gamma^\alpha[\tfrac{1}{2}(1-\gamma_5)]s\}\{\bar{\mu}\gamma_\alpha[\tfrac{1}{2}(1+\gamma_5)]e\}$$

with the suppression factor

$$\epsilon \simeq \sin\theta_c\cos\theta_c\sin\varphi\cos\varphi\frac{\Delta m_q^2\Delta m_N^2\zeta}{M_W^2(m_q^2+m_N^2)}, \quad (5)$$

where m_q and m_N are the (average) quark and heavy-lepton masses, respectively, and $\Delta m_q^2 = m_c^2 - m_u^2$; ζ is a factor of order unity.[8] This result is to be compared with the familiar $2W$-exchange amplitude in $K_L\to\bar{\mu}\mu$ decay[7]:

$$T_{\bar{d}s\underset{2W}{\to}\bar{\mu}\mu} \simeq \frac{g^4}{64\pi^2}\frac{\epsilon_0}{M_W^2} \times\{\bar{d}\gamma^\alpha[\tfrac{1}{2}(1-\gamma_5)]s\}\{\bar{\mu}\gamma_\alpha[\tfrac{1}{2}(1-\gamma_5)]\mu\},$$

with

$$\epsilon_0 \approx \sin\theta_c\cos\theta_c\frac{\Delta m_q^2}{M_W^2}\left[\ln\left(\frac{M_W^2}{m_q^2}\right)-1\right]. \quad (6)$$

We note that the suppression factors in Eqs. (5) and (6) both have factors of M_W^{-2} and are roughly of the same order of magnitude. *Because of the GIM mechanism, the cancellation between N-lepton propagators does not introduce further severe suppression factors.*[9] Keeping in mind terms like $(\cos\varphi\sin\varphi)^2$ and the fact that $2W$ contribution to $K_L\to\mu\mu$ is slightly smaller than the 2γ diagrams, we estimate the branching ratio $\Gamma_{K_L\to e\mu}/\Gamma_K \simeq 10^{-2}\Gamma_{K_L\to\mu\mu}/\Gamma_K \simeq 10^{-10}$.[10]

The effective quark interaction for $K^+\to\pi^+e\mu$ is the same as Eq. (5). Putting it between the hadronic states of K and π, we can relate it to the K_{l3} form factors.[7] This way we obtain

$$\frac{\Gamma_{K^+\to\pi^+e^+\mu^-}}{\Gamma_{K^+\to\pi^0e^+\nu}} \simeq \left(\frac{g^2}{16\pi^2}\frac{\epsilon}{\sin\theta_c}\right)^2 \simeq 10^{-10}, \quad (7)$$

which leads to a branching ratio of $\Gamma_{K\to\pi e\mu}/\Gamma_{K^+} \simeq 10^{-11}$.

Our results, together with present experimental limits, are summarized in Table I. These numerical estimates are given purely for illustrative purposes; they are clearly sensitive to the precise values of the mixing angle and heavy-lepton masses.

Neutrino oscillations.—Neutrino oscillations can take place if separate conservation of μ- and

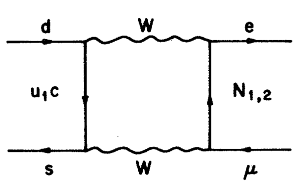

FIG. 2. Free-quark diagrams for the $K\to e\mu$ decay in a theory with $V+A$ currents.

TABLE I. Summary of our results vs experimental limits. All the branching ratios are proportional to $(\sin\varphi\cos\varphi\,\Delta m_N^2)^2$ which, for order-of-magnitude estimates, we took to be 1 GeV^4.

Process	Our estimates	Experimental bound
$\mu\to e\gamma$	10^{-10}	$<2.2\times10^{-8}$ (Ref. 4)
$\mu\to3e$	10^{-12}	$<6\times10^{-9}$ (Ref. 11)
$K_L\to e\mu$	10^{-10}	$<1.6\times10^{-9}$ (Ref. 12)
$K^\pm\to\pi^\pm e\mu$	10^{-11}	$<1.4\times10^{-8}$ (Ref. 13)

e-lepton numbers does not hold *and* if neutrinos have different masses.[14] We shall present here an attempt to estimate the oscillation length: $l_{\rm osc} \simeq 4\pi p_\nu/\Delta m_\nu^2$ (where p_ν is the neutrino beam momentum and Δm_ν^2 is the difference between the squares of the neutrino masses).

The mass scale of elementary fermions seems to be on the order of 1 GeV as given by the heavy quarks and heavy leptons. The neutrino masses are so extremely small that we find it hard to believe that it is not zero in some sense. We shall assume that they are massless in the absence of weak interactions. In the usual $V-A$ theory [Eq. (1)], the neutrino will stay massless to all orders; hence, there will be no oscillation. But in many theories with both $V-A$ and $V+A$ currents, higher-order weak processes will naturally bring about their masses and mixings.[15,16]

Again, to be specific, consider the theory in Eq. (2) together with

$$\begin{pmatrix} N_1(\theta) \\ M_1 \end{pmatrix}_L \quad \begin{pmatrix} N_2(\theta) \\ M_2 \end{pmatrix}_L \quad \begin{pmatrix} \nu_1 \\ M_1 \end{pmatrix}_R \quad \begin{pmatrix} \nu_2 \\ M_2 \end{pmatrix}_R . \qquad (8)$$

The mass (or mixing) terms have helicity structure of the left-right type. The lowest-possible-order contributions to neutrino masses and mixings are the two-loop diagrams. We can then make an order-of-magnitude estimate of the general neutrino-mass matrix element

$$m_\nu \simeq \left(\frac{\alpha}{\pi} m_\mu \ln\frac{\Lambda^2}{M_W^2}\right) \frac{\zeta(\theta,\varphi)}{m_N} \left(\frac{\alpha}{\pi} m_M \ln\frac{\Lambda^2}{M_W^2}\right) \simeq 10^{-1}\,\mathrm{eV}. \qquad (9)$$

The numerical result is based on the estimates $\zeta(\theta,\varphi) \simeq 10^{-1}$ and $m_M/m_N \simeq 1$, and an "educated" guess[17] of $\ln(\Lambda^2/M_W^2) \simeq 10^{-1}$. Thus for a 40-GeV neutrino beam, the oscillation length will be about 10^4 km.

Details of our calculations, more precise discussion of the theoretical assumptions, and extensions to theories with Majorana particles will be presented in a forthcoming paper.[18]

We would like to thank Professor Lincoln Wolfenstein for helpful discussions and continual encouragement.

Note added.—J. D. Bjorken, K. Lane, and S. Weinberg in a forthcoming paper make the following very interesting observation about the model in Eq. (2). If the charged-lepton masses arise from gauge-invariant bare terms and/or the vacuum expectation value of the Higgs scalar singlet, there will be uniquely determined mixings among the left-handed neutrinos and heavy leptons. A set of "left-right" diagrams similar to Fig. 1 for the $\mu \to e\gamma$ process then exists. These new diagrams increase the decay rate in Eq. (4) by a factor of 25. Our independent computation agrees with this conclusion. We thank these authors for communicating their results to us prior to publication. We are also grateful to F. Wilczek and A. Zee for pointing out an error in the original manuscript of this paper.

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[1] F. Wilczek and A. Zee [Nucl. Phys. B 106, 461 (1976)] have considered, in gauge theories, the radiative transitions of new leptons to an electron or a muon $U \to l\gamma$.

[2] Incidentally, this theory would not allow parity nonconservation in atomic physics since the neutral current coupling to the electron is purely vector. $N_i(\varphi)$ denote similar combinations as $\nu_i(\theta)$ in (1).

[3] From the lower limit of neutrino oscillation length, one can deduce that $\Delta m_\nu^2 \lesssim 25$ eV^2 (see A. K. Mann and H. Primakoff, to be published).

[4] S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. 133, B768 (1964).

[5] This cancellation only requires that e and μ couple to independent fields. The proof is straightforward and will be given by Cheng and Li, to be published.

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[7] M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974); M. K. Gaillard, B. W. Lee, and R. Shrock, Phys. Rev. D 13, 2674 (1976).

[8] Detailed expression will be given by Cheng and Li, Ref. 5.

[9] This is reminiscent of the GIM cancellation in $K_L \to 2\gamma$ (see Ref. 7), and is related to the number of W propagators in the diagram.

[10] The branching ratio of $K_L \to \mu\mu$ is 10^{-8} [see W. C. Carithers *et al.*, Phys. Rev. Lett. 30, 1336 (1973), and 31, 1025 (1973); Y. Fukushima *et al.*, Phys. Rev. Lett. 36, 348 (1976)].

[11] S. M. Korenchenko *et al.*, Yad. Fiz. 13, 1265 (1971) [Sov. J. Nucl. Phys. 13, 728 (1971)].

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[15] T. P. Cheng, unpublished, and Phys. Rev. D 14, 1367 (1976).

[16] H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1975).

[17]In Ref. 15, a model with an enlarged gauge group is constructed in which all such "self-energy" diagrams are finite. The result is, to a good approximation, equivalent to cutting off the divergence in the SU(2) ⊗ U(1) theory with a mass comparable to M_W.

[18]Cheng and Li, Ref. 5.

Effect of Inelastic Excitation on Elastic Scattering of Heavy Ions

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The elastic-scattering angular distribution of 90-MeV ^{18}O on ^{184}W exhibits a dramatic deviation from the typical Fresnel shape, which cannot be reproduced by standard optical-model calculations. The effect, a decrease in the elastic cross section below the Rutherford cross section even at forward angles, is due primarily to Coulomb excitation, and will be even more pronounced for heavier projectiles.

Recent analyses of two-nucleon transfer reactions induced by heavy ions on deformed rare-earth targets have shown the necessity of explicitly including the strong coupling within the ground-state rotational band (GSB) in order to reproduce the observed angular distributions.[1] The present experiment was undertaken to look for evidence of this strong coupling in the elastic-scattering angular distributions.

The typical angular distribution for heavy-ion elastic scattering has a form characteristic of Fresnel diffraction (see, e.g., the ^{18}O + ^{208}Pb data in Fig. 1). Most previous measurements of heavy-ion "elastic scattering" from deformed targets have obtained similar angular distributions, but have suffered from energy resolution insufficient to resolve low-lying members of the GSB from the ground state.[2] Recently, a survey of elastic scattering of ^{12}C ions from various Nd isotopes,[3] in which the 2^+ states were resolved, revealed a systematic damping of the oscillations in the ratio $\sigma_{el}/\sigma_{Ruth}$ with increasing target deformation. The angular distributions were still of the Fresnel diffraction form, however. In this Letter we demonstrate that, for projectiles with sufficiently large Z, inelastic excitation can drastically modify the shape of the elastic-scattering angular distribution, even at angles well forward of the grazing angle.

The experiments were performed with 70-MeV ^{12}C and 90-MeV ^{18}O beams from the Brookhaven National Laboratory (BNL) tandem Van de Graaff facility. Great care was taken to minimize motion of the beam spot on the target since elastic scattering can be quite sensitive to small variations in scattering angle. The beam position was monitored by two silicon surface-barrier detec-

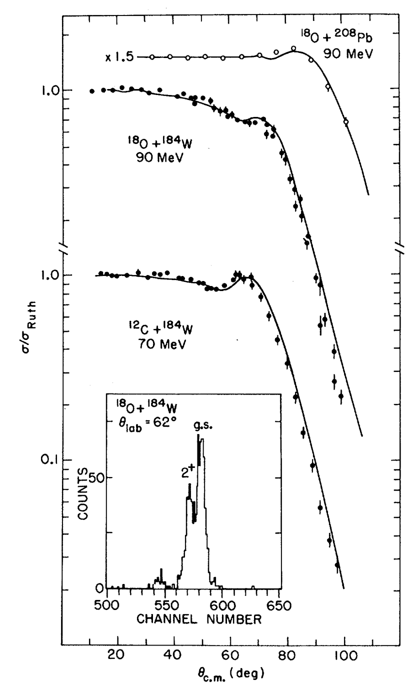

FIG. 1. Elastic-scattering angular distributions. Error bars represent random errors arising from statistics and uncertainties in fitting to the peaks. The curves are coupled-channels calculations using the parameters of Table I. Inset: A typical position spectrum from the focal-plane detector, showing the clear resolution of the ground and first excited states in ^{184}W.